Evaluating consumer damages from tying in multi-sided digital markets*

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Abstract

Based on real-world competition cases, this paper examines the impact on consumers' welfare from tying practices by a dominant firm in multi-sided digital markets with network economies and imperfect competition in the retail market. We show that tying is profitable, always decreases consumer surplus, and increases retail prices. However, the retail price of the tying firm increases less, and hence its market share rises. Inspired by the Google Android case, the theoretical model is calibrated with real-world data to illustrate the quantification of inflicted consumer harm, estimated to be roughly between \$5 and \$11 per handset purchased worldwide. Given the approach followed in the model, this should be considered a conservative estimation of the actual harm.

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1 Introduction

The “Google Android case”, an example of the abuse of dominant position in multi-sided markets through tying, has received considerable attention from the media, courts, and academics. As defined by Tirole (2005), tying strategies occur when the sale of one product (the tying product) is conditional on the purchase of another product (the tied product). This may lead to anti-competitive outcomes due to the risk of harming (and even excluding) rival producers in potentially competitive market segments and, ultimately, harming customers.

Through its Decision dated July 18, 2018 (Case AT.40099, Google Android), confirmed in the 2022 General Court ruling, the European Commission concludes that Google has violated Article 102 of the Treaty on the Functioning of the European Union (TFEU) and Article 54 of the European Economic Area (EEA) Agreement by tying its search engine (Google Search) to its application store (Google Play Store) and its web browser, Google Chrome, to both the Google Play Store and Google Search through contractual agreements with Original Equipment Manufacturers (OEMs) and Mobile Network Operators (MNOs). This infringement began in January 2011. The agreements involved, known as Mobile Application Distribution Agreements (MADAs), Anti-Fragmentation Agreements (AFAs), and Revenue Share Agreements (RSAs), excluded other Android-based app stores and competitors of proprietary Google apps (including Google Search) from accessing Android devices. These agreements aimed to ensure the exclusive pre-installation of Google’s search engine (SE) as the default SE on Android devices, thereby protecting, reinforcing, and leveraging Google’s dominance in the digital advertising market for search, which accounts for nearly 90% of Google’s revenue. These practices effectively foreclosed potentially more efficient search engines and app stores, while hindering the growth of "forked" Android devices, produced by OEMs using the Android operating system but lacking the Google Play Store and other essential Google apps. This could potentially reduce the choices available to consumers, limit consumer surplus, and stifle innovation. A record fine of 4.343 billion euros was imposed in Europe in 2018, which was reduced to 4.125 billion euros in 2022. Despite this unprecedented amount, some authors question whether these fines are adequate to deter large platforms that generate massive annual revenues (e.g., Cahill and Wang, 2024).

More specifically, Google's infringement, as a "gatekeeper" in the terminology of the European Union Digital Markets Act, consists of four separate violations: (i) tying the Google Search app to the Play Store through restrictions outlined in the MADAs, which required OEMs to pre-install Google Search to obtain a license for using the Play Store; (ii) tying Google Chrome to the Play Store and Google Search through restrictions outlined in the MADAs, requiring OEMs to pre-install Google Chrome (the browser app) in order to obtain a license for using the Play Store; (iii) licensing the Play Store and Google Search conditioned on the anti-fragmentation obligations in the AFAs, which stipulated that OEMs wishing to pre-install Google apps could not sell devices (the forked devices) running versions of Android not approved by Google (they could only sell compatible devices that do not threaten Google's position); and (iv) granting revenue share payments, such as ad profits, to OEMs and MNOs on the condition that they did not pre-install any competing search engine on any device within an agreed portfolio (RSAs). Google was also accused of violating Article 101 of the TFEU through its agreements.

The exclusionary practices of Google limit opportunities for competing apps to gain visibility, reinforcing Google's dominance. A behavioral remedy was imposed, with the European Commission ordering Google to provide a choice screen on Android devices, instead of having a pre-installed search engine. This decision was later revised to allow participation in the choice screen at no cost, as Google had previously charged its rivals a fee for their display.[1] To date, Google has faced the highest number of antitrust investigations in Europe, has received the two largest fines, and has also been the target of the most significant number of antitrust actions across digital firms worldwide

[1] See Ganesh (2024) for an analysis of remedies in digital markets, where the author also proposes more radical approaches, such as subsidizing the next best competitor.

(Radivojević et al., 2024).

Google's ecosystem, which includes a bundle of complementary services (sometimes referred to as Google suite) like Gmail, Google Maps, YouTube, Google Drive, and many others in addition to Google's Play Store, is built on the Android operating system (OS). The Android OS holds approximately 70% market share in the operating systems sector. Unlike Apple's iOS, the other major player, Android is a licensable OS. Any OEM can develop a new open-source version of Android, essentially creating a forked Android. In the retail market, device prices and OEM margins are largely determined by these manufacturers, with limited competition and influenced by agreements established with software providers.

The Google Play Store is a dominant player in the market for software and utilities that run on the Android OS. Although it initially lagged Apple's App Store, it is now the app store with the largest number of apps and the highest number of users worldwide. In the market for Android app stores, its market share approaches nearly 100%. This scenario highlights the significant role of network effects, both direct (within the group) and indirect (across groups).[2] Such network benefits usually result in economic advantages for larger players as they increase consumers' willingness to pay for products with larger user bases.

Google also holds a dominant position in the national markets for search services (different languages imply a definition of the relevant geographical market at the national level). Across all 30 national markets of the EEA analyzed in the Decision, Google had an average market share exceeding 94% in 2016.

Since the foundational work of Katz and Shapiro (1985) on network externalities, and with the advent of the internet in the late 20th century, the literature on two-sided markets has gained

[2]Karhu et al. (2024) provide an interesting and comprehensive characterization of positive and negative network externalities in multi-sided platforms, applying it to the Google Android case, which involves four distinct sides: advertisers, hardware manufacturers, users, and developers.

significant and increasing attention, as digital markets grow larger and larger. From the late 1990s to the early 2000s, many works on platform economics were published, some inspired by real-world cases in these giants tech industry.

One such work is Etro and Cafarra (2017), who identify mechanisms through which Google may extract additional profits by tying its search engine to the Google Suite. In particular, the authors provide a context in which they demonstrate how contractual agreements such as MADAs, RSAs, and AFAs may serve as tools to ensure the exclusive pre-installation of Google's SE, preventing the emergence of "forked" Androids. The authors illustrate that Google may be using RSAs to outbid rival search engines in negotiations with OEMs regarding the specifications of their smartphones, specifically concerning the default menu of apps offered to customers when they purchase a smartphone, thereby blocking their entry, which ultimately harms customers by precluding access to alternative (possibly better) search engines. Note that consumers are assumed to single home.[3] To assess the welfare and consumer surplus effects of tying, the authors compare two settings: (i) the no-tying scenario, in which Google provides the Google Play Store (GPS) as a standalone product and the rival firm offers the high-quality search engine, as a counter-factual; (ii) the tying scenario, in which Google links its search engine to the GPS so that two types of equipment are presented to customers: the Google device, which comes with pre-installed Google Search and GPS, and a forked Android device, featuring a superior search engine but lacking an app store (or a lesser option, with no net utility to customers, when comparing the benefits of using it to the costs, which may arise, for example, from differences in usability). The authors demonstrate that tying may deter a rival SE from entering the market (if the additional surplus from GPS more than compensates for the quality differential in the search engines). Moreover, they find that RSAs do not pay for exclusivity (which may be granted by MADAs). Instead, they offer effective side payments to OEMs to ensure

[3] For the effects of tying under multi-homing see, for instance, Choi (2010).

that they accept the tying product rather than supporting the "forked" Android. When examining the welfare and consumer surplus outcomes, the authors indicate that tying is detrimental to both welfare and consumer surplus: on the one hand, it results in the consumption of an inferior search engine variant; on the other hand, it may lead to higher device prices.

Another article inspired by the Google-Android case that examines the incentives for a monopolist to tie its product to another product in a two-sided market, where competition could occur without tying, is Choi and Jeon (2021).The authors demonstrate that, under nonnegative price constraints, the "One Monopoly Profit Argument" may be invalid.[4] Indeed, in digital markets, committing to negative prices (or prices below marginal cost, which is typically close to zero) may be more challenging because consumers can, with the click of a button, download an app to receive the payment and then not use it at all. This means that negative pricing does not necessarily increase demand, rendering it ineffective as a mechanism for credibly enhancing cross-network effects.[5,6] This is an important point, as the authors show that the inability to set negative prices ultimately limits competition in the tied good market (which also implies that firms may extract extra profits when employing tying). Consequently, the authors argue that tying may be profitable in two-sided markets with constraints on setting negative prices, offering a microfounded model to justify Google's strategy of tying its search engine with the Google Play Store (and Google Chrome).

[4] The argument of consumer harm had been previously challenged by the Chicago School (Director and Levi, 1956 and Stigler, 1963) under the so called "One Monopoly Profit Argument", which pointed out that tying "is not a rational method of obtaining a second source of monopoly profits, because an increase in the price charged for the tied product will, as a first approximation, reduce the price that the purchaser is willing to pay for the tying product" (Posner, 1978). Thus, the Chicago School claimed that tying would not raise antitrust issues as it would not bring extra profits to firms engaged in such strategies. This argument has become quite influential in the first tying anti-trust cases (Abbott and Wright, 2009).

[5] Hahn and Yoon (2020) and Hahn et al. (2023) investigate tying in two-sided markets when below-cost or negative pricing is allowed. They also examine a situation in which a monopolist in the tying market may compete with a more efficient rival in the tied market segment. Both market segments are two-sided markets, and there are two groups of customers, who differ in their view regarding the relations between the goods (a group considers the two goods independent, whereas the other group views them as complements). The existence of these two groups is shown to be a necessary condition for profitable tying to emerge when we relax the restrictions of non-negative prices imposed in Choi and Jeon (2021). Profitable tying often reduces consumer surplus and social welfare.

[6] Another significant contribution addressing the competitive effects of tying in two-sided markets with non-negative prices is Amelio and Jullien (2012), who argue that tying may benefit consumers when it increases participation on both sides of the market.

The authors also examine the welfare and consumer surplus effects of tying in this context. They find that in a model with homogeneous consumers, tying is welfare-reducing due to inefficiency in product allocation in the tied market. Nevertheless, the authors contend that consumers may not be worse off, as they are compensated by a reduction in the price of the tied combination, which offsets the consumption of the low-quality variant (resulting from tying).[7] A dimension in which Etro and Cafarra (2017) differ from Choi and Jeon (2021) is in assuming that the tying and tied software are not sold to final consumers but, rather, to OEMs, allowing for the possibility of negative pricing. This occurs when platforms pay OEMs to be installed on their equipment, which, in turn, leads to lower equipment prices for final users.

An important limitation of the Etro and Cafarra (2017) model is that it does not consider network effects and, therefore, misses a key ingredient of multi-sided markets, which could affect conclusions in the sense that tying might have the economic benefit of bringing more customers together, thus magnifying overall network effects. However, tying may also influence the equilibrium price structure in a way that ultimately hurts customers, so the interplay between tying and network effects remains unclear. Furthermore, most Android markets worldwide are dominated by a few brands, making the perfect competition assumption in Etro and Cafarra (2017) not very realistic. The current paper attempts to fill these two gaps by extending the Etro and Cafarra (2017) setting to: i) account for the possibility of network effects; ii) provide a more realistic view of OEM competition by assuming an oligopoly scenario, in which OEMs may have some market power (as is the case with tech giants such as Samsung). This allows us to conclude that their theory of harm remains valid when we extend the baseline model along these two lines: network effects and market power at the OEM level.

[7] Choi et al. (2024) extend Choi and Jeon (2021) in order to provide a new rationale for tying associated with network effects. The authors demonstrate how firms may use tying to lock in high-end customers and maximize network benefits, which may then lead to the eviction of rival firms (or block their entry) even when they are more efficient.

We solve the game under tying and the counterfactual scenario of no tying. In the absence of tying, we assume that Google sells the Google Suite separately as a standalone product through OEMs. This situation aligns with the theoretical concept of the absence of tying, and preserves the OEMs' intermediation role, serving as a point of comparison with the tying scenario. In Appendix B, we also present a no-tying scenario where Google continues to offer the Google suite for free, reflecting the prevailing situation prior to tying. Clearly, this scenario leads to significantly higher damages for consumers. We demonstrate that tying is profitable, always reduces consumer surplus, and raises device retail prices. However, it increases more the price of the competing search engine, which results in an increase in Google's market share.[8] We also derive the fundamental expression for the change in consumer surplus as a function of relevant parameters such as Google's market share, consumers' valuation of the Play Store, advertising revenues, and the magnitude of network effects, with the aim of assessing the harm to consumers resulting from Google's restrictive practices.

As OEMs benefit from selling differentiated products, one OEM may prefer to have the Google Play Store installed, while another may not (the latter can be viewed as a "forked" Android). This differentiation softens price competition between the two OEMs. In the absence of tying, the search engines offered by Google and its rival hold equal value for consumers, leading OEMs to choose the one whose developer made the highest bid. This scenario would create intense competition on these bids, reducing the marginal cost of handsets and lowering prices for consumers. Conversely, with tying, the OEM aiming to differentiate by not including the Play Store will have no choice but to select the rival's search engine. This situation enables Google's rival to make a lower bid, which raises the marginal costs for this OEM and, in turn, the price it sets for final consumers. As prices

[8] One of the first papers to examine the circumstances under which firms may benefit from tying strategies, even when rivals remain active in the market, is Choi (2010). The author investigates tying in two-sided markets and raises the point that tying might improve welfare under multi-homing assumptions. De Corniere and Taylor (2021) also find a rationale for profitable bundling in a vertical value chain that shares some similarities with Google's setting. However, they do not explicitly consider the two-sided market nature of the industry.

are strategic complements, all handset prices increase with tying. In our model, tying is not an exclusionary practice (a case in which the harm from consumer surplus variation would be greater), since competition is not eliminated; actually, rival's profits do even increase.

At the end of the paper we illustrate how the results of our theoretical model can be used to obtain estimates of welfare and consumers' losses resulting from tying practices in search engine markets. To this end, we calibrate the analytical expression of the change in consumer surplus by using observed values at the world level. This illustrative application allows us to approximately estimate, in monetary terms, the damage to consumers resulting from Google's anti-competitive practices. An interval of [$5.40, $10.92] per handset sold worldwide is obtained under conservative assumptions. It is acknowledged that, in legal investigation, calibration can be pertinent to competition cases. However, it should be noted that, like any model, our framework is a stylized representation of reality, facing the usual trade-off between accuracy and simplicity; therefore, it does not fully capture all the market features and details involved. Approximations are inherently imperfect, so a conservative approach should be adopted and is indeed followed throughout the paper in all modeling decisions made. Thus, the estimated harm can be considered a lower bound of the actual damage.

The remainder of the paper is structured as follows. The next section solves the model under tying and no tying. Section 3 calibrates the model and estimates the reduction in consumer surplus associated with tying. Section 4 concludes. Appendix A contains formal proofs, and Appendix B presents an "alternative" formulation to the no tying scenario.

2 The model

2.1 Main assumptions

We assume that two OEMs, denoted by OEM 1 and OEM 2, produce hardware equipment (smartphones or tablets) at constant marginal cost C. To be of any value, this hardware needs to have several types of software installed or pre-installed. These include, among others, the operating system, a search engine, some pre-installed apps and an app store. To simplify, we consider that OEMs require only two types of software, which we denote by X and Y. This software is developed by two software developers, firm G and firm E (respectively a Giant-tech and an entrant). Both firms develop competing versions of software Y (say, search engines), which are equally valued by consumers,[9] but firm G is the only (monopolist) developer of software X (say, play store). The marginal costs of these firms are assumed to be zero.

Firm G may tie the sale of software Y (the tied software) to the sale of software X (the tying software). In order to have the software installed, the two firms set a unit price (B_G and B_E, which may be negative) that the OEMs may accept or reject. A negative price corresponds to a payment made by G or E to OEMs. We call these price offers "bids", to differentiate them from the retail prices (P_1 and P_2) that OEM's set for final users.

Final users' valuation for the equipment depends on what is installed. Any hardware needs to have some version of software Y installed. Any such version is equally valued by final users at R.[10] This value is common to all users and assumed to be sufficiently high so that all purchase the hardware. However, not all users value software X (the tying monopoly software) equally. We denote a given user valuation for software X by R_X and assume that $R_X := R_x + \delta' D_X$, where: (i)

[9] If the search engine of the entrant was of higher quality, as in Etro and Cafarra (2017), consumer harm from tying would be even higher than our estimates because, as we will see, tying increases the market share of G.

[10] As far as consumer damages are concerned, this is a conservative assumption when compared to the alternative assumption of a higher willingness to pay for firm E's software.

R_x follows a uniform distribution in interval $[0, V]$ and (ii) $\delta' D_X$ reflects the existence of a network externality with respect to software X: the higher the number of users of this software, D_X, the higher the valuation that each user has for it. This externality can both be direct or indirect. The number of users affect each user's willingness to pay because a higher number of users implies that there are more users to interact with or because it leads to a higher number of app developers who prefer to be present in app stores with more users (a feature that is not included explicitly in the model). The total number of users is denoted by Θ.

In addition to the revenue (if any) from allowing OEMs to install their software, the two software firms also obtain some revenue from advertisements places in their software. We denote by A firm G's advertising profit per user of software X and by B the advertising profit per user of software Y received by both firm G and firm E.

The timing of the game is the following. Initially, firm G commits to tie or not the two products. Then, firm G and firm E simultaneously set the prices charged (or paid to) the OEMs to include their software. Then, if tying does not take place, firm G sets the standalone price for software X, which we denote by Q. Finally, the two OEMs simultaneously set their retail prices and consumers make their choices.

In what follows it will be convenient to present all these variables and parameters in a normalized way, that is, divided by V. The normalization of variables B_G, B_E, Q, P_1 and P_2 is respectively denoted by b_G, b_E, q, p_1 and p_2. Parameters C, R, R_x, A and B, when normalized, are denoted by c, r, r_x, α and β. Finally, we write $\delta'\Theta/V$ as δ, firm i's profit, Π_i, is denoted in its normalized form by $\pi_i = \Pi_i/(\Theta V)$, and consumer surplus, CS, is denoted also in its normalized form by $cs = CS/(\Theta V)$.

The following table presents all the model endogenous variables and parameters.

Endogenous variables		
Bid by software developer $j = G, E$	B_j	$b_j = B_j/V$
OEM i retail price, $i = 1, 2$	P_j	$p_j = P_j/V$
Number of users of software X	D_X	-
Standalone price for software X	Q	$q = Q/V$
Firm i's profit	Π_i	$\pi_i = \Pi_i/(\Theta V)$
Consumer surplus	CS	$cs = CS/(\Theta V)$
Consumer indifferent between the OEMs	$\widetilde{R}_x$	
Parameters		
OEM's marginal cost	C	$c = C/V$
Willingness to pay for software Y	R	$r = R/V$
Willingness to pay for software X	$R_X = R_x + \delta' D_X$	$r_X = R_X/V$
Network effects parameter	δ'	$\delta = \delta'\Theta/V$
Total number of users	Θ	-
Firm G's profit per user of sofware X	A	$\alpha = A/V$
Firm G's and firm E's profit per user of software Y	B	$\beta = B/V$

The next section presents the model equilibrium in the presence of tying. This is followed by a section describing the equilibrium without tying and by another section in which the effects of tying on consumer welfare are discussed.

2.2 Tying

Assume that firm G has tied software X and Y, committing not to sell both separately or as standalone.

2.2.1 Demand faced by OEMs.

The demand for the OEM's hardware depends on whether both OEMs chose firm G or not. If both selected firm G as their software supplier, then users will simply select the cheapest equipment available.[11] If one OEM selected firm G but the other selected firm E, we assume without loss of generality that OEM 1 was the firm selecting firm G. Then, for equal prices all users will purchase from OEM 1 but, if OEM 2's price is lower, some users (those with a relatively low valuation for software X) will purchase from OEM 2. The demand for OEM i is denoted by D_i and, given the assumption that all users purchase some equipment, we have that $D_2 = \Theta - D_1$. The following Lemma presents the demand faced by both OEMs in terms of the normalized prices.

Lemma 1:

a) If both OEM 1 and OEM 2 chose firm G (or firm E), the demand for OEM 1 is:

$$D_1(p_1, p_2) = \begin{cases} \Theta & p_1 < p_2 \\ \frac{\Theta}{2} \quad \text{if} & p_1 = p_2 \\ 0 & p_1 > p_2 \end{cases}$$

[11] We assume that in case of equal prices 50% of the users will select each OEM.

b) If OEM 1 chose firm G and OEM 2 chose firm E, the demand for OEM 1 is:

$$D_1(p_1,p_2) = \begin{cases} \Theta & p_1 < p_2 + \delta \\ \frac{1-p_1+p_2}{1-\delta}\Theta & \text{if} \quad p_2 + \delta < p_1 < p_2 + 1 \\ 0 & p_1 > p_2 + 1 \end{cases}$$

■

Network externalities (δ) increase the demand for OEM 1 and decrease the demand for OEM 2. For the quantity demanded to be decreasing in own price, we assume that $1 - \delta > 0$ or $\delta < 1$.

Given these demands, the normalized profit of the two OEMs, if different software is installed, is given by

$$\pi_1 = (p_1 - c - b_G)\frac{1 - p_1 + p_2}{1 - \delta}$$
$$\pi_2 = (p_2 - c - b_E)\left(1 - \frac{1 - p_1 + p_2}{1 - \delta}\right)$$

2.2.2 OEM price decision

The following Lemma presents the OEM's equilibrium prices.

Lemma 2: a) If both OEM 1 and OEM 2 chose firm i, the equilibrium prices are

$$p_1 = p_2 = c + b_i$$

b) If OEM 1 chose firm G and OEM 2 chose firm E, the equilibrium prices in an interior solution, that is, if $2\delta - 1 < b_G - b_E < 2 - \delta$, are

$$p_1 = c + \frac{2}{3}b_G + \frac{1}{3}b_E + \frac{2}{3} - \frac{1}{3}\delta$$
$$p_2 = c + \frac{1}{3}b_G + \frac{2}{3}b_E + \frac{1}{3} - \frac{2}{3}\delta$$

where both software developers are active. ■

The retail price of OEM 1 increases with its marginal cost $c+b_G$ and also, but to a lesser extent, with the rival's marginal cost $c+b_E$. The network externality parameter δ creates downward pressure on prices. This parameter decreases the demand for OEM 2, leading this firm to set lower prices. As prices are strategic complements, OEM 1's equilibrium price also decreases. The corresponding equilibrium demand for OEM 1 is

$$D_1(p_1, p_2) = \frac{1 - p_1 + p_2}{1 - \delta}\Theta = \frac{1}{3}\frac{2 - \delta - b_G + b_E}{1 - \delta}\Theta$$

and the OEMs' normalized profits are

$$\pi_1 = \frac{1}{9}\frac{(2 - \delta - b_G + b_E)^2}{1 - \delta}$$
$$\pi_2 = \frac{1}{9}\frac{(1 - 2\delta + b_G - b_E)^2}{1 - \delta}$$

2.2.3 OEM software choice

The following Lemma presents the equilibrium software selection by the OEMs.

Lemma 3: In equilibrium, one OEM will choose firm G and the other OEM will choose firm E. ■

To avoid fierce price competition, the two OEMs choose to differentiate their product by selecting a different software supplier. Without loss of generality we assume OEM 1 selects firm G.

Given this decision, firm G and firm E's normalized profits are

$$\pi_G = (\alpha + \beta + b_G)\,\frac{1}{3}\frac{2-\delta-b_G+b_E}{1-\delta}$$
$$\pi_E = (\beta + b_E)\left(1-\frac{1}{3}\frac{2-\delta-b_G+b_E}{1-\delta}\right)$$

2.2.4 Firm G and firm E's bidding decision

The following Lemma presents the equilibrium bids made by firms G and E to the OEM's.

Lemma 4: The equilibrium bids are

$$b_G = \frac{5}{3} - \frac{2}{3}\alpha - \beta - \frac{4}{3}\delta$$
$$b_E = \frac{4}{3} - \frac{1}{3}\alpha - \beta - \frac{5}{3}\delta$$

■

The difference in bids has an uncertain sign:

$$b_G - b_E = \frac{1}{3}\left(1 - \alpha + \delta\right)$$

The reason for the uncertain sign is the following. On the one hand, firm G has an additional incentive to lower b_G which is the increase in the number of users (i.e, the number of users who purchase from OEM 1) because of the revenue per user generated by software X, denoted by α. On the other hand, firm E faces a lower demand when compared to firm G because of the relatively lower valuation for OEM 2's hardware that lacks software X.

Proposition 1: Let $\delta < 4/5$ and $\alpha < 4 - 5\delta$. In equilibrium, if firm G ties the sale of software

X and Y:

a) Retail prices are

$$
\begin{aligned}
p_1 &= c - \beta - \frac{5}{9}\alpha - \frac{16}{9}\delta + \frac{20}{9} \\
p_2 &= c - \beta - \frac{4}{9}\alpha - \frac{20}{9}\delta + \frac{16}{9}
\end{aligned}
$$

b) The consumer indifferent between the two OEM's is

$$
\widetilde{R}_x = \frac{1}{9}\frac{4 - 5\delta - \alpha}{1 - \delta}V
$$

c) Normalized profits are

$$
\begin{aligned}
\pi_G &= \frac{1}{27}\frac{(5 + \alpha - 4\delta)^2}{1 - \delta}, \ \pi_E = \frac{1}{27}\frac{(4 - \alpha - 5\delta)^2}{1 - \delta} \\
\pi_1 &= \frac{1}{81}\frac{(5 + \alpha - 4\delta)^2}{1 - \delta}, \ \pi_2 = \frac{1}{81}\frac{(4 - \alpha - 5\delta)^2}{1 - \delta}
\end{aligned}
$$

d) Normalized consumer surplus is

$$
cs = r - c + \beta + \frac{360\delta^3 + 8\delta^2 (9\alpha - 124) - 8\delta (19\alpha - 112) + \alpha^2 + 82\alpha - 263}{162 (1 - \delta)^2}
$$

■

OEM 1 will set a higher equilibrium price

$$
p_1 - p_2 = \frac{1}{9}(4 - \alpha + 4\delta) > 0
$$

which reflects the fact that it has installed software X, thus placing in the market a more complete product.

2.3 No Tying

Consider now that firm G sells software X as standalone. The demand faced by the OEM's is the same as in Lemma 1 because consumers are not affected by whether a given OEM installed software X as a standalone or as a tied product.

2.3.1 OEM price decision

The following Lemma presents the OEM's equilibrium prices in this case.

Lemma 5:

a) If both OEM 1 and OEM 2 chose software Y from firm i, with bid b_i, and if both (or none) accepted the bid q for software X, then equilibrium prices are:

$$p_1 = p_2 = c + b_i + q$$

with $q = 0$ in case no OEM accepted q.

b) If OEM 1 accepted $b_i < b_j$, OEM 2 accepted bid $b_j > b_i$, and both (or none) accepted the bid q for software X, then equilibrium prices are:

$$p_1 = c + q + b_j - \varepsilon$$

$$p_2 = c + q + b_j$$

where $\varepsilon \to 0$ and $q = 0$ in case no OEM accepted q.

c) If OEM 1 accepted b_i, OEM 2 accepted bid b_j and only OEM 1 accepted the bid q for software

X, then equilibrium prices are:

$$p_1 = c + \frac{2}{3}(q + b_i) + \frac{1}{3}b_j + \frac{2}{3} - \frac{1}{3}\delta$$
$$p_2 = c + \frac{1}{3}(q + b_i) + \frac{2}{3}b_j + \frac{1}{3} - \frac{2}{3}\delta$$

■

As can easily be seen by comparing the prices in Lemma 5 c) with those in Lemma 2, there is no difference at this stage due to tying when only one OEM installs software X. The only difference is that the bid for software X under no tying is explicitly presented, instead of being included in the bid for software Y.

2.3.2 OEM software choice

The following Lemma presents the equilibrium software selection by the OEMs.

Lemma 6: In equilibrium, both OEMs will select the lowest bid for software Y and only one will accept the bid for software X. ■

To avoid fierce price competition, the two OEMs choose to differentiate their product by selecting a different software supplier. Without loss of generality, in what follows we assume OEM 1 accepts q. Note also that tying does not affect decisions at this stage.

2.3.3 Firm G and firm E's bidding decision

As OEMs accept the lowest bid, we will have OEMs setting equilibrium prices (from Lemma 5 c), with $b_i = b_j = b$) given by

$$
\begin{aligned}
p_1 &= c + b + \frac{2}{3}q - \frac{1}{3}\delta + \frac{2}{3} \\
p_2 &= c + b + \frac{1}{3}q - \frac{2}{3}\delta + \frac{1}{3}
\end{aligned}
$$

Given these prices, users selecting OEM 1 are $D_1 = \frac{1}{3}\frac{2-q-\delta}{1-\delta}\Theta$ and firm G and firm E's normalized profits are:

$$
\pi_G = \begin{cases} (\beta + b_G)\,\Theta + (\alpha + q)D_1 & b_G < b_E \\ 0 + (\alpha + q)D_1 \qquad \text{if} & b_G > b_E \\ (\beta + b_G)\,\frac{\Theta}{2} + (\alpha + q)D_1 & b_G = b_E \end{cases}
$$

$$
\pi_E = \begin{cases} 0 & b_G < b_E \\ (\beta + b_E)\,\Theta \quad \text{if} & b_G > b_E \\ (\beta + b_E)\,\frac{\Theta}{2} & b_G = b_E \end{cases}
$$

The following Lemma presents the equilibrium bids.

Lemma 7: If there is no tying, the equilibrium bids are:

$$
b_E = b_G = -\beta
$$

$$
q = \begin{cases} 1 - \frac{1}{2}(\alpha + \delta) & \alpha \leq 4 - 5\delta \\ 2\delta - 1 & \alpha > 4 - 5\delta \end{cases} \quad \text{if}
$$

■

The following Proposition summarizes the equilibrium in the absence of tying.

Proposition 2: Let $\delta < 4/5$ and $\alpha < 4 - 5\delta$. In equilibrium, if firm G does not tie the sale of software X and Y:

a) Retail prices are

$$p_1 = c - \beta + \frac{4}{3} - \frac{1}{3}\alpha - \frac{2}{3}\delta$$
$$p_2 = c - \beta + \frac{2}{3} - \frac{1}{6}\alpha - \frac{5}{6}\delta$$

b) The consumer indifferent between the two OEM's is

$$\widetilde{R}_x := \frac{1}{6}\frac{4 - \alpha - 5\delta}{1 - \delta}V$$

c) Normalized profits are

$$\pi_G = \frac{1}{12}\frac{(\alpha - \delta + 2)^2}{1 - \delta},\ \pi_E = 0$$
$$\pi_1 = \frac{1}{36}\frac{(\alpha - \delta + 2)^2}{1 - \delta},\ \pi_2 = \frac{1}{36}\frac{(\alpha - 3\delta)^2}{1 - \delta}$$

d) Normalized consumer surplus is

$$cs = r - c + \beta + \frac{60\delta^3 + \delta^2(12\alpha - 167) - \delta(26\alpha - 152) + \alpha^2 + 16\alpha - 44}{72(1 - \delta)^2}$$

■

2.3.4 The effects of tying

As mentioned throughout the previous sections, the effects of tying arise through the bids received by OEMs. Therefore, we now compare the total bids accepted by OEM 1 and OEM 2 with those in the case of tying. OEM 2 (the OEM that does not accept the bid for software X) now receives a bid of $-\beta$ whereas before, with tying, the bid was $b_E = -\beta + \frac{1}{3}(4 - 5\delta - \alpha)$. The bid with no tying is lower (or OEM 2's cost is higher). The aggregate bids accepted by OEM 1 (the OEM that also accepts the bid for software X) are now

$$1 - \frac{1}{2}(\alpha + \delta) - \beta$$

whereas with tying these were $b_G = \frac{5}{3} - \frac{2}{3}\alpha - \beta - \frac{4}{3}\delta$. Again, the bid with no tying is lower:

$$1 - \frac{1}{2}(\alpha + \delta) - \beta - (\frac{5}{3} - \frac{2}{3}\alpha - \beta - \frac{4}{3}\delta) = -\frac{1}{6}(4 - \alpha - 5\delta) < 0$$

By tying, firm G is able to offer a software "package" that is differentiated from firm E's. Without tying, the competition to sell software Y is very intense and this benefits both OEM's, in particular OEM 2 who does not purchase software X. OEM 1 will purchase software X at the monopoly price, but the overall effect is still positive with this OEM ending up with lower marginal cost than in the case of tying, where it is the exclusive supplier of software X.

Observing firm G's best-response in the case of tying is revealing of the effects involved. In the case of tying, firm G's best response is (see proof of Lemma 4):

$$b_G = \frac{1}{2}(2 - \alpha - \beta - \delta + b_E)$$

If firm E sets $b_E = -\beta$, the optimal b_G would be

$$b_G = \frac{1}{2}\left(2 - \alpha - \beta - \delta - \beta\right) = 1 - \frac{1}{2}\left(\alpha + \delta\right) - \beta$$

This is exactly the sum of firm G's bids for software X and Y in the absence of tying. Thus, it is clear that the effects of tying on bids (and hence on retail prices and final user choices) result from the increase in the equilibrium b_E. Firm E makes lower bids (or charges a higher price to allow its software to be installed) which will lead to lower equilibrium bids by both firms, as bids are strategic complements. This increases the costs of both OEMs and leads to higher retail prices, which harms customers. As the costs of OEM 1 increase less that those of OEM 2, so do the retail prices. This leads firm G and OEM 1 to increase their market share with tying.

The next Proposition summarizes the effects of tying.

Proposition 3: Let $\delta < 4/5$ and $\alpha < 4 - 5\delta$. Then, tying: i) is profitable, ii) always decreases consumer surplus, iii) increases both OEM retail prices, but increases more p_2 than p_1, and iv) the market share of firm G and OEM 1 increase. ■

Corollary 1: The loss in consumer surplus decreases with α and with δ. ■

Thus, the introduction of δ in the model is, when comparing to Etro and Cafarra (2017), a conservative approach as far the consumer's loss is concerned.

The change in consumer surplus can be divided in three terms, all of them negative:

i) Users such that $0 < R_x < \frac{1}{9}\frac{4-\alpha-5\delta}{1-\delta}V$ will always purchase from OEM 2. They are impacted by tying due to the increase in P_2. The change in their surplus is

$$\frac{\Delta CS_I}{\Theta V} = -\frac{5}{162}\frac{\left(\alpha + 5\delta - 4\right)^2}{1-\delta} < 0$$

ii) Users such that $\frac{1}{9}\frac{4-\alpha-5\delta}{1-\delta}V < R_x < \frac{1}{6}\frac{4-\alpha-5\delta}{1-\delta}V$ will purchase from OEM 1 if there is tying but from OEM 2 if there is not. The change in their surplus is

$$\frac{\Delta CS_{II}}{\Theta V} = (10\delta - 9)\,\frac{(\alpha + 5\delta - 4)^2}{648\,(1-\delta)^2} < 0$$

iii) Users such that $\frac{1}{6}\frac{4-\alpha-5\delta}{1-\delta}V < R_x < V$ will always purchase from OEM 1. They are impacted by tying due to the increase in P_1 but also due to the increase in network externalities. The change in their surplus is

$$\frac{\Delta CS_{III}}{\Theta V} = -\frac{1}{108}\,(4-5\delta)\,(2+\alpha-\delta)\,\frac{4-\alpha-5\delta}{(1-\delta)^2} < 0$$

3 Estimating the reduction in consumer surplus

The type of economic modelling of the industry addressed in the previous section is useful to identify the effects involved and to present theoretical motives for tying to lead to an increase in profits at the cost of a reduction in consumer surplus. In this case, tying reduces the intensity of the price competition between software developers. The model can also be used, given its assumptions, to estimate the magnitude of the effects involved in a specific case, although this is not its main purpose and should be considered as a mere illustration.

In this section, we illustrate how our stylized model can be used to obtain an estimate of the consumer harm inflicted worldwide, through the above mentioned mechanism, in the "Google Android" case. To avoid using data from multiple sources, we focus on the period 2016-2022. Firm G in the model is obviously Google, software Y corresponds to a search engine and a browser, and software X to Google's Play Store. Proposition 3 shows that tying the two types of software always reduces consumer surplus. In order to estimate how much consumers are affected by tying, it is necessary to calibrate the model, that is, to identify values for the model parameters such that the equilibrium of the model (its output) replicates observed industry features. We use the tying case as the factual (observed) scenario to calibrate the model. Naturally, the non tying (non observed) case is our counter factual scenario that we take as benchmark when assessing anti-competitive effects of tying for consumers.

3.1 Model Calibration

The change in consumer surplus per handset sold is given by

$$\frac{\Delta CS}{\Theta} = (4 - \alpha - 5\delta)\,\frac{5\alpha + 349\delta - 180\delta^2 - 164}{648\,(1-\delta)^2}V$$

Thus, to obtain an estimate of the magnitude of the consumer loss, $\frac{-\Delta CS}{\Theta}$, one needs to estimate values for parameters α, δ and V. With industry data publicly available online one can obtain the approximate value of firm G's market share, which is denoted by s_G, and also of firm G's advertising profit per user from selling software X, which is denoted by A.

By definition, $\alpha = A/V$ and as the predicted market share, $\widetilde{R}_x/V$, depends on α and δ, one can make the observed values equal to those predicted by the model, that is

$$s_G = \frac{1}{9}\frac{\alpha - 4\delta + 5}{1-\delta}$$

$$A = \alpha V$$

in order to obtain an estimate of the unobserved parameter values as a function of the observed values s_G and A:

$$V = \frac{A}{4\delta - 5 + 9s_G\,(1-\delta)}$$

$$\alpha = 4\delta - 5 + 9s_G\,(1-\delta)$$

where $4\delta - 5 + 9s_G(1-\delta) > 0$ or $\delta < \frac{5-9s_G}{4-9s_G}$ when $s_G > \frac{5}{9}$, as is the case.

Plugging these values in the expression for the change in consumer surplus we obtain

$$\frac{\Delta CS}{\Theta} = \frac{1}{8}A(1-s_G)\frac{20\delta + 5s_G - 21}{4\delta - 5 + 9s_G(1-\delta)}$$

Even the "observed" values for s_G and A, based on the data available, may differ between sources. Thus, it is relevant to know how the selection of different "observed" variables affects the estimated change in consumer surplus. If $s_G > \frac{1}{2}$, we have that:

Lemma 8:

i) A higher A increases the estimated consumer loss;

ii) A lower s_G increases the estimated consumer loss.

iii) A higher δ increases the estimated consumer loss.

So, conservative estimations of consumer loss have low A and δ and high s_G.

A relevant consideration has to do with the time period the estimated losses refer to. The relevant demand in the model is a demand for handsets with installed software. All prices are also prices per handset sold or bids to install software per handset. Thus, the loss in consumer surplus is a loss per handset purchased.

3.1.1 Firm G's market share, s_G

In the model, there is a single market share for firm G. It corresponds to the fraction of users that purchase the equipment with firm G 's tied software installed (the search engine, play store and browser). In reality, there are different market shares according to the product considered. This happens because some users may change the software that was pre-installed, because market shares may be measured in terms of the number of queries (for search engines) or of the number of downloads (for app stores). However, to calibrate the model, one needs to have a single value for

s_G. We use as a proxy for Google's market share the average of the market shares available. As the model includes only Android (or forked Android) equipment, we exclude the market share related to other operating systems (Apple, Nokia, Blackberry) and average the monthly values for the market shares of Google Search Engine and Chrome for the relevant periods. The average montly market share of Google Search (in mobile devices worldwide) between 2016 and 2022 was 95%, the average market share of Chrome (plus Android) between 2016 and 2022 (excluding browsers for other operating systems) was 74.5%. We take the simple average of these two market shares to calibrate the model. Hence, we consider $s_G = \frac{95+74.5}{2} = 84.75\%$.[12]

Another alternative would be to use only the lowest of the two market shares, reflecting the idea that the market share of the bundle equals the market share of the component with the lowest market share. If, say, 70% of the users have software X from firm G and 60% have software Y from the same firm, then the number of consumers who have both is likely to be 60%. As mentioned above (Lemma 8), the use of a high market share decreases the estimated consumer loss, so the option for the average market share in detriment of the minimum is a conservative approach to consumers' harm.

3.1.2 Revenue per handset, A

In order to obtain a value for A, the profit from Google Play Store for each handset, we divide yearly Google play revenue by the total number of active Android users.[13] We then convert all values to 2022 using the illustrative interest rate of 4%. Table 1 reports these values.[14] The average revenue

[12] Source: https://gs.statcounter.com/search-engine-market-share/mobile/worldwide/#monthly-201601-202212 and https://gs.statcounter.com/browser-market-share/mobile/worldwide/#monthly-201601-202212

[13] We approximate profit by revenue, under the assumption that the cost of an extra user (marginal cost) is zero.

[14] The data sources are:
https://prioridata.com/data/google-play-revenue-statistics/
https://scoop.market.us/android-phones-statistics/

per year is placed at 13.666 (measured in 2022 USD).

Year	Play Store Revenue	Android users	Revenue per user	
	($bn)	(bn)	($)	(2022 $)
2016	15	1.7	8.824	11.165
2017	21.21	2	10.605	12.903
2018	24.82	2.3	10.791	12.624
2019	30.63	2.5	12.252	13.782
2020	38.62	2.8	13.793	14.919
2021	47.94	3	15.98	16.619
2022	42.32	3.1	13.652	13.652

This revenue per user per year is multipled by the average number of years that a smartphone is used. Some sources place the worldwide lifetime of a smartphone at 28.1 months for 2016 and others present estimates for 2024 at 4.2 years.[15] We take the average of the two: 3.25 years. Hence, $A = 3.25 * 13.666 = 44.415$.

This is also a conservative estimate because there are other tied products in addition to the app store that also generate revenue (or reduce costs) (recall from Lemma 8 that the estimated consumer loss increases with A). For instance, one more user of Chrome (also tied) will save Google some money by reducing the payment it makes to other browsers in order to have its search engine as the default one in the browser's main page.

[15] See https://www.statista.com/statistics/786876/replacement-cycle-length-of-smartphones-worldwide/ and https://www.gartner.com/en/documents/4887431

3.1.3 Network externalities parameter, δ

There is no clear empirical counterpart for this parameter. Therefore, we will present the estimate as a function of δ. Although theoretically δ could be as large as $\frac{5-9s_G}{4-9s_G} = 0.72433$, we consider that very high values of δ may not make economic sense. The reason is the following. In addition to the market share, the model presents other values that depend on these parameters: V and d. In particular, the price difference d between OEM 1 and OEM 2 cannot be very large, as OEM 2 should have a positive margin. This price difference is

$$d = P_1 - P_2 = \frac{1}{9}\left(4\left(\delta+1\right)-\alpha\right)V = -\frac{8883}{200}\frac{339\delta+61}{1451\delta-1051}$$

Assuming a value of \$254 for the average price of a handset,[16] it is hard to conceive that the price difference could exceed (or even be close to) \$100. This allows us to put an upper bound on δ:

$$-\frac{8883}{200}\frac{339\delta+61}{1451\delta-1051} < 100 \Leftrightarrow \delta < 0.639$$

3.2 Consumer surplus losses

The following figure represents the loss per handset, $-\frac{\Delta CS}{\Theta}$, as a function of δ and for $s_G = 84.75\%$ and $A = 44.415$.

[16] https://scoop.market.us/android-phones-statistics/

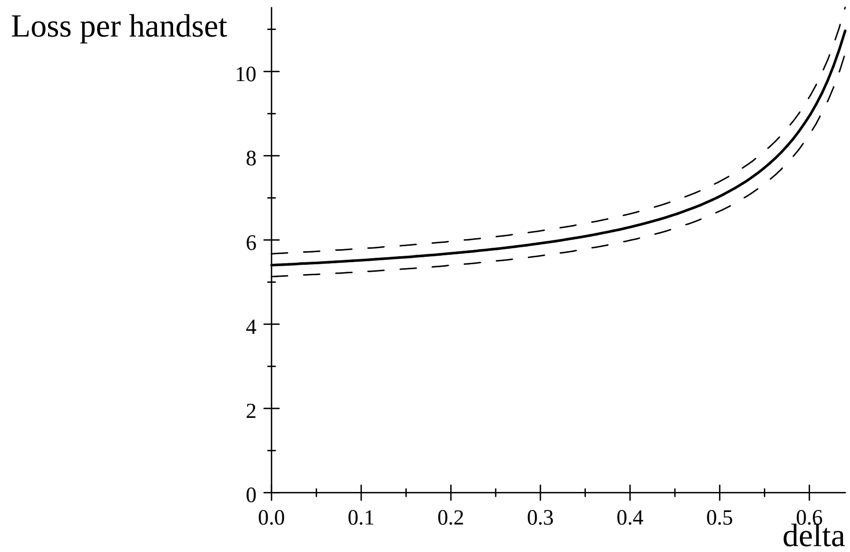

Figure 1: Loss per handset as a function of δ.

The two dashed lines correspond to the effect on consumer surplus loss when the value of A is allowed to increase or decrease by 5%.

Taking the upper limit on δ into consideration, the consumer damage, per handset, lies in the interval $[\$5.40, \$10.92]$. The following figure presents d and $V/2$ as a function of δ. If estimates for the price difference of handsets with and without the tied products installed were available (d), and/or if the willingness to pay for software X of the median consumer was available ($V/2$), these could be used to approximate the value for δ that is consistent with the observed value(s) and thus fine tune the interval of admissible values for δ. In its absence, however, the interval for δ cannot be made more precise.

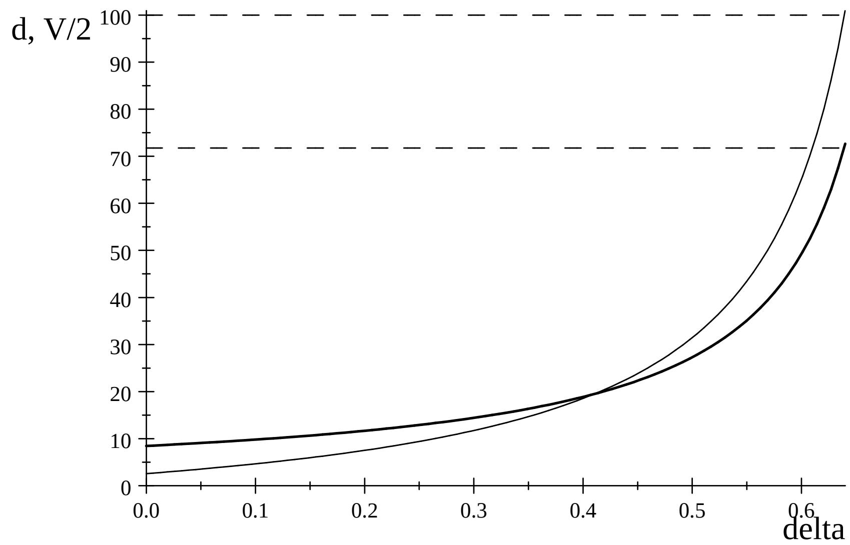

Figure 2: Median consumer willingness to pay for software X, $V/2$, (thicker curve) and price difference between OEM 1 and OEM 2, d, as a function of δ.

The tying practice increases prices and firm G's market share by:

$$\Delta P_1 = \frac{541863}{100}\frac{\delta - 1}{1451.0\delta - 1051}$$

$$\Delta P_2 = \frac{541863}{80}\frac{\delta - 1.0}{1451\delta - 1051}$$

$$-\frac{\Delta \widetilde{R}_x}{V} = 0.076$$

Google's market share is thus estimated to rise 7.6%. Estimated price increases range between \$5.15 and \$15.80 for the handset sold by OEM 1, and between \$6.44 and \$19.75 for the handset sold by OEM 2. The following figure presents the change in prices due to tying, which is closely related to the consumer loss.

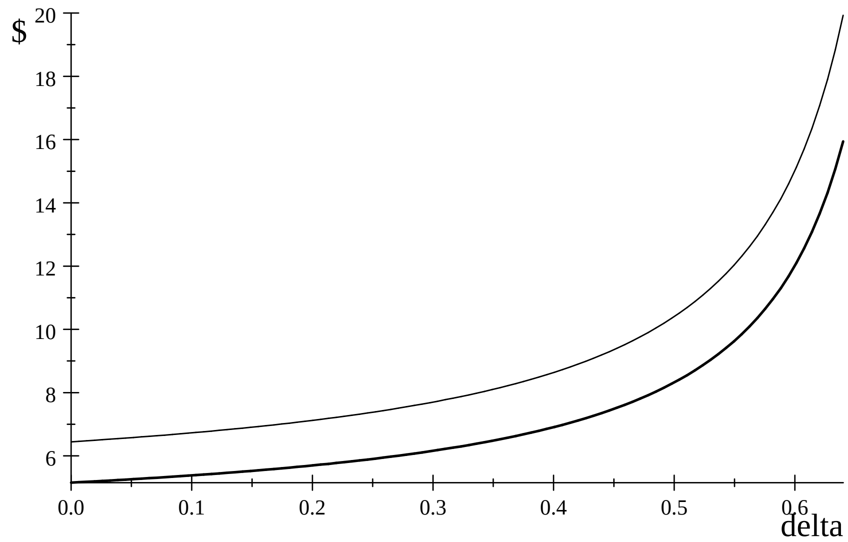

Figure 3: Change in handset prices due to tying as a function of δ (ΔP_1 corresponds to the thicker curve).

4 Conclusion

Although the rise of the internet era and the digital economy were expected to foster more competition due to lower entry barriers and reduced switching costs between rival companies, pushing markets toward the perfect competition paradigm, the reality has instead shown the emergence of highly concentrated markets dominated by tech giants, who resort to anticompetitive practices such as tying, to consolidate and extend their market power, leveraging on network effects and interconnected demands.

Tying occurs when a firm conditions the sale of the tied product on the acquisition of the tying product. This practice is anti-competitive and can potentially harm consumers. Based on the "Google Android case", this paper investigates the impact on consumer surplus of tying Google Search to the Google Play Store and Google Chrome to Google Search and Google Play Store, following the European Commission's Decision.

Our theoretical model extends Etro and Cafarra (2017) to a more realistic framework, considering network externalities in the use of the Play Store and imperfect competition between OEMs that assemble hardware and software and sell the devices to final consumers. In the absence of tying, the search engine offered by Google and the rival's search engine hold the same value for consumers, leading OEMs to select the one whose developer makes the highest bid. This creates strong competition on these bids, which lowers the marginal cost of handsets and the prices for consumers. Conversely, with tying, the OEM that aims to differentiate by not having the Play Store installed will have no choice but to opt for the rival's search engine. This situation allows Google's rival to submit a lower bid, increasing the marginal costs for this OEM and subsequently raising the price it charges to final consumers. As prices are strategic complements, all handset prices rise with tying, resulting in a consumer loss. We demonstrate that tying is profitable (even without excluding the rival), decreases consumer surplus, and raises the rival's price more than that of Google's equipped device, thereby increasing Google's market share.

We adopt a conservative approach to assessing consumer loss. Specifically, and unlike Etro and Cafarra (2017), our model acknowledges that the two competing search engines are equally valued by consumers. Furthermore, when calibrating the model with real-world industry data (market share, revenues from the Play Store, and others), we always choose the most conservative estimates. This involves minimizing consumer loss to establish a lower bound for the damage. The estimated damage per handset purchased worldwide, which ranges roughly from \$5 to \$11, should therefore be viewed as an approximation of the true harm, considering the simplifying nature of models and the assumptions made. Our calibration exercise demonstrates the practical applicability of the theoretical model in quantifying real-world damages arising from tying practices and indicates its potential utility in similar cases.

Appendix A

This appendix contains the proofs of the results in the main text.

Proof of Lemma 1:

a) If both OEMs install the same software, users will simply select the OEM with the lowest price.

b) If OEM 1 installs firm G's tied software and OEM 2 installs firm E's software, then the users that will select OEM 1's equipment are those such that

$$R + R_x + \delta' E(D_X) - P_1 > R - P_2 \Leftrightarrow R_x > \widetilde{R}_x := P_1 - P_2 - \delta' E(D_X)$$

where $E(D_X)$ is the users expectation of the demand for software X and $\widetilde{R}_x$ denotes the user indifferent between the two alternatives. The number of such users is

$$D_1 = \frac{V - (P_1 - P_2 - \delta' E(D_X))}{V}\Theta$$

Assuming rational expectations, $E(D_X) = D_1$ and

$$D_1 = \frac{V - (P_1 - P_2 - \delta' D_1)}{V}\Theta \Leftrightarrow D_1 = \frac{V - P_1 + P_2}{V - \Theta\delta'}\Theta = \frac{1 - p_1 + p_2}{1 - \delta}\Theta$$

The user indifferent between the two OEMs is then

$$\widetilde{R}_x := P_1 - P_2 - \delta' E(D_X) = P_1 - P_2 - \delta' \frac{V - P_1 + P_2}{V - \Theta\delta'}\Theta$$
$$= V\frac{P_1 - P_2 - \Theta\delta'}{V - \Theta\delta'} = V\frac{p_1 - p_2 - \delta}{1 - \delta}$$

if $\widetilde{R}_x < V$, that is, if $p_1 < p_2 + 1$, and if $\widetilde{R}_x > 0$, that is, if $p_1 > p_2 + \delta$. ■

Proof of Lemma 2:

a) Follows from the equality between price and marginal cost when OEM's are not differentiated.

b) First-order conditions are

$$\frac{\partial\left((p_1-c-b_G)\frac{1-p_1+p_2}{1-\delta}\right)}{\partial p_1} = \frac{c-2p_1+p_2+b_G+1}{1-\delta} = 0$$
$$\frac{\partial\left((p_2-c-b_E)\left(1-\frac{1-p_1+p_2}{1-\delta}\right)\right)}{\partial p_2} = \frac{c-\delta+p_1-2p_2+b_E}{1-\delta} = 0$$

that result in the following best response functions

$$p_1 = \frac{1}{2}(c+b_G+1+p_2)$$
$$p_2 = \frac{1}{2}(c+b_E-\delta+p_1)$$

from where the result is obtained. Second-order conditions are verified. Note that an interior solution is reached if

$$\widetilde{R}_x := V\frac{p_1-p_2-\delta}{1-\delta} = \frac{1}{3}V\frac{1-2\delta+b_G-b_E}{1-\delta}$$

belongs to $[0,V]$, that is, if $2\delta-1<b_G-b_E<2-\delta$. ■

Proof of Lemma 3: Given the equilibrium prices determined in Lemma 2, at this stage OEM's play the following game:

		OEM 2 Choose firm G	OEM 2 Choose firm E
OEM 1	Choose firm G	$0,0$	$\frac{1}{9}\frac{(2-\delta-b_G+b_E)^2}{1-\delta},\frac{1}{9}\frac{(1-2\delta+b_G-b_E)^2}{1-\delta}$
	Choose firm E	$\frac{1}{9}\frac{(1-2\delta+b_G-b_E)^2}{1-\delta},\frac{1}{9}\frac{(2-\delta-b_G+b_E)^2}{1-\delta}$	$0,0$

where there are two asymmetric Nash equilibria, in which OEM's choose a different software supplier. ■

Proof of Lemma 4: First-order conditions are

$$\frac{\partial\left((\alpha+\beta+b_G)\frac{1}{3}\frac{2-\delta-b_G+b_E}{1-\delta}\right)}{\partial b_G}=\frac{1}{3}\frac{2-\alpha-\beta-\delta-2b_G+b_E}{1-\delta}=0$$
$$\frac{\partial\left((\beta+b_E)\left(1-\frac{1}{3}\frac{2-\delta-b_G+b_E}{1-\delta}\right)\right)}{\partial b_E}=\frac{1}{3}\frac{1-\beta-2\delta+b_G-2b_E}{1-\delta}=0$$

that result in the following best response functions

$$b_G=\frac{1}{2}\left(2-\alpha-\beta-\delta+b_E\right)$$
$$b_E=\frac{1}{2}\left(1-\beta-2\delta+b_G\right)$$

from where the result is obtained. Second-order conditions are verified. Note that an interior solution is reached if $4\delta-5<0<\alpha<4-5\delta$ and that OEM 1's equilibrium demand is

$$D_1(p_1,p_2)=\frac{1-p_1+p_2}{1-\delta}\Theta=\frac{1}{3}\frac{2-\delta-b_G+b_E}{1-\delta}\Theta=\frac{1}{9}\frac{\alpha-4\delta+5}{1-\delta}\Theta$$

■

Proof of Proposition 1:

The proof of a), b) and c) follows directly from Lemmas 1 to 4. Consumer surplus in equilibrium is given by

$$CS=\int_0^{\frac{1}{9}\frac{4-\alpha-5\delta}{1-\delta}V}(R-P_2)\frac{\Theta}{V}dR_x+\int_{\frac{1}{9}\frac{4-\alpha-5\delta}{1-\delta}V}^{V}(R+R_x+\delta'\frac{1}{9}\frac{\alpha-4\delta+5}{1-\delta}\Theta-P_1)\frac{\Theta}{V}dR_x\Leftrightarrow$$
$$\frac{CS}{V\Theta}=\int_0^{\frac{1}{9}\frac{4-\alpha-5\delta}{1-\delta}V}(r-p_2)\frac{1}{V}dR_x+\int_{\frac{1}{9}\frac{4-\alpha-5\delta}{1-\delta}V}^{V}(r+\frac{R_x}{V}+\delta\frac{1}{9}\frac{\alpha-4\delta+5}{1-\delta}-p_1)\frac{1}{V}dR_x$$

Evaluating this expression at the equilibrium prices yields the result. ■

Proof of Lemma 5:

Part a) follows from the fact that OEM's sell homogeneous products, have symmetric costs and compete on price. Corresponding profits are $\pi_1 = \pi_2 = 0$. Part b) follows from the fact that OEM's sell homogeneous products, have asymmetric costs (with the costs of OEM 1 lower than those of OEM 2) and compete on price. Corresponding profits are $\pi_1 = b_j - b_i$ and $\pi_2 = 0$.

c) Given that the demands are as in Lemma 1 b), normalized profits are now

$$\begin{aligned}
\pi_1 &= (p_1 - c - b_i - q)\frac{1 - p_1 + p_2}{1-\delta} \\
\pi_2 &= (p_2 - c - b_j)\left(1 - \frac{1 - p_1 + p_2}{1-\delta}\right)
\end{aligned}$$

First-order conditions are

$$\begin{aligned}
\frac{\partial\left((p_1 - c - b_i - q)\frac{1-p_1+p_2}{1-\delta}\right)}{\partial p_1} &= \frac{1 + c + q + b_i - 2p_1 + p_2}{1-\delta} = 0 \\
\frac{\partial\left((p_2 - c - b_j)\left(1 - \frac{1-p_1+p_2}{1-\delta}\right)\right)}{\partial p_2} &= \frac{c + b_j - \delta + p_1 - 2p_2}{1-\delta} = 0
\end{aligned}$$

that result in the following best response functions

$$\begin{aligned}
p_1 &= \frac{1}{2}(c + b_i + q + 1 + p_2) \\
p_2 &= \frac{1}{2}(c + b_j - \delta + p_1)
\end{aligned}$$

from where the result is obtained. Second-order conditions are verified. Note that an interior solution is reached if

$$\widetilde{R}_x := \frac{1}{3}\frac{1 + q - 2\delta + b_i - b_j}{1-\delta}V$$

belongs to $[0, V]$, that is, if $2\delta - 1 - q < b_i - b_j < 2 - \delta - q$. The corresponding profits are

$$\pi_1 = \frac{1}{9}\frac{(2 - q - \delta - b_i + b_j)^2}{1 - \delta}$$
$$\pi_2 = \frac{1}{9}\frac{(1 - q - 2\delta + b_i - b_j)^2}{1 - \delta}$$

■

Proof of Lemma 6: Given the equilibrium prices determined in Lemma 5, OEM's play the following game with respect to accepting or rejecting bid q. Below it is assumed that OEM 1 has accepted bid $b_i \leq b_j$ and OEM 2 has accepted bid b_j. The profits are presented in the proof of Lemma 5.

		OEM 2: Accept q	OEM 2: Reject q
OEM 1	Accept q	$b_j - b_i, 0$	$\frac{1}{9}\frac{(2-q-\delta-b_i+b_j)^2}{1-\delta}, \frac{1}{9}\frac{(1-q-2\delta+b_i-b_j)^2}{1-\delta}$
	Reject q	$\frac{1}{9}\frac{(1-q-2\delta+b_j-b_i)^2}{1-\delta}, \frac{1}{9}\frac{(2-q-\delta-b_j+b_i)^2}{1-\delta}$	$b_j - b_i, 0$

If, alternatively, OEM 2 had accepted the lowest bid b_i the game would be

		OEM 2: Accept q	OEM 2: Reject q
OEM 1	Accept q	$0, 0$	$\frac{1}{9}\frac{(2-q-\delta)^2}{1-\delta}, \frac{1}{9}\frac{(1-q-2\delta)^2}{1-\delta}$
	Reject q	$\frac{1}{9}\frac{(1-q-2\delta)^2}{1-\delta}, \frac{1}{9}\frac{(2-q-\delta)^2}{1-\delta}$	$0, 0$

We have that $\frac{1}{9}\frac{(2-q-\delta)^2}{1-\delta} \geq \frac{1}{9}\frac{(2-q-\delta-b_j+b_i)^2}{1-\delta}$ and $\frac{1}{9}\frac{(1-q-2\delta)^2}{1-\delta} \geq \frac{1}{9}\frac{(1-q-2\delta+b_i-b_j)^2}{1-\delta}$. Indeed,

$$\frac{1}{9}\frac{(2-q-\delta)^2}{1-\delta} - \frac{1}{9}\frac{(2-q-\delta+b_i-b_j)^2}{1-\delta} = \frac{1}{9}(b_j-b_i)\frac{2(2-q-\delta)+b_i-b_j}{1-\delta} > 0$$
$$\frac{1}{9}\frac{(1-q-2\delta)^2}{1-\delta} - \frac{1}{9}\frac{(1-q-2\delta+b_i-b_j)^2}{1-\delta} = \frac{1}{9}(b_j-b_i)\frac{2(1-q-2\delta)+b_i-b_j}{1-\delta} > 0$$

If only the lowest bid is accepted, then OEMs play the following game with respect to accepting or rejecting bid q

		OEM 2	
		Accept q	Reject q
OEM 1	Accept q	$0, 0$	$\frac{1}{9}\frac{(2-q-\delta)^2}{1-\delta}, \frac{1}{9}\frac{(1-q-2\delta)^2}{1-\delta}$
	Reject q	$\frac{1}{9}\frac{(1-q-2\delta)^2}{1-\delta}, \frac{1}{9}\frac{(2-q-\delta)^2}{1-\delta}$	$0, 0$

where there are two asymmetric Nash equilibria, in which OEMs choose a different software supplier.■

Proof of Lemma 7: As can be seen, the best response of firm E is to undercut firm G until $b_E = -\beta$. Firm G's best response function is also to undercut b_E until $b_G = -\beta$. The optimal q given that both OEMs will choose the lowest price (highest bid) is the one that maximizes

$$(\alpha+q)D_1 = (\alpha+q)\frac{1}{3}\frac{2-q-\delta}{1-\delta}\Theta$$

subject to $\frac{1}{3}\frac{2-q-\delta}{1-\delta}\Theta \leq \Theta \Leftrightarrow q \geq 2\delta - 1$. First-order conditions are

$$\frac{\partial\left((\alpha+q)\frac{1}{3}\frac{2-q-\delta}{1-\delta}\Theta\right)}{\partial q} = \frac{1}{3}\Theta\frac{2-2q-\alpha-\delta}{1-\delta} = 0$$
$$q = \frac{1}{2}(2-\alpha-\delta)$$

The constraint is not binding if $\frac{1}{2}(2-\alpha-\delta) \geq 2\delta - 1 \Leftrightarrow \alpha \leq 4 - 5\delta$. Otherwise, $q = 2\delta - 1$. ■

Proof of Proposition 2:

The proof of a), b) and c) follows directly from Lemmas 5 to 7. Consumer surplus in equilibrium is given by

$$CS = \int_0^{\frac{1}{6}\frac{4-\alpha-5\delta}{1-\delta}V} (R-P_2)\frac{\Theta}{V}dR_x + \int_{\frac{1}{6}\frac{4-\alpha-5\delta}{1-\delta}V}^{V} (R+R_x+\delta'\left(\frac{V-\frac{1}{6}\frac{4-\alpha-5\delta}{1-\delta}V}{V}\right)\Theta - P_1)\frac{\Theta}{V}dR_x \Leftrightarrow$$

$$\frac{CS}{V\Theta} = \int_0^{\frac{1}{6}\frac{4-\alpha-5\delta}{1-\delta}V} (r-p_2)\frac{1}{V}dR_x + \int_{\frac{1}{6}\frac{4-\alpha-5\delta}{1-\delta}V}^{V} (r+\frac{R_x}{V}+\frac{1}{6}\delta\frac{\alpha-\delta+2}{1-\delta}-p_1)\frac{1}{V}dR_x$$

Evaluating this expression at the equilibrium prices yields the result. ■

Proof of Proposition 3:

Recall that in order for an interior equilibrium to exist, $\alpha+5\delta-4<0$. A necessary condition for this to be possible is that $\delta<\frac{4}{5}$.

i) Tying is profitable if and only if

$$\frac{1}{27}\frac{(\alpha-4\delta+5)^2}{1-\delta} > \frac{1}{12}\frac{(\alpha-\delta+2)^2}{1-\delta} \Leftrightarrow -(5\alpha+16-11\delta)\frac{\alpha+5\delta-4}{108(1-\delta)} > 0$$

ii) Tying decreases consumer surplus if and only if

$$\Delta cs = r-c+\beta+\frac{360\delta^3+8\delta^2(9\alpha-124)-8\delta(19\alpha-112)+\alpha^2+82\alpha-263}{162(1-\delta)^2} -$$
$$\left(r-c+\beta+\frac{60\delta^3+\delta^2(12\alpha-167)-\delta(26\alpha-152)+\alpha^2+16\alpha-44}{72(1-\delta)^2}\right)$$
$$=(4-\alpha-5\delta)\frac{5\alpha+349\delta-180\delta^2-164}{648(1-\delta)^2}<0$$

This is negative if $\alpha<36\delta^2-\frac{349}{5}\delta+\frac{164}{5}$, which is always true because $\alpha<4-5\delta$ and $4-5\delta<36\delta^2-\frac{349}{5}\delta+\frac{164}{5}\Leftrightarrow\frac{36}{5}(5\delta-4)(1-\delta)<0$.

iii) Tying increases prices if and only if

$$\Delta p_1 = c - \beta - \frac{5}{9}\alpha - \frac{16}{9}\delta + \frac{20}{9} - (c - \beta + \frac{4}{3} - \frac{1}{3}\alpha - \frac{2}{3}\delta) = -\frac{4}{18}(\alpha + 5\delta - 4) > 0$$
$$\Delta p_2 = c - \beta - \frac{4}{9}\alpha - \frac{20}{9}\delta + \frac{16}{9} - (c - \beta + \frac{2}{3} - \frac{1}{6}\alpha - \frac{5}{6}\delta) = -\frac{5}{18}(\alpha + 5\delta - 4) > 0$$

which is always true.

Tying increases more p_2 than p_1:

$$\Delta p_1 - \Delta p_2 = -\frac{4}{18}(\alpha + 5\delta - 4) - \left(-\frac{5}{18}(\alpha + 5\delta - 4)\right) = \frac{\alpha + 5\delta - 4}{18} < 0$$

iv) Tying increases the market share of firm G :

$$\frac{V - \Delta\widetilde{R}_x}{V} = \frac{V - (\frac{1}{9}\frac{4-5\delta-\alpha}{1-\delta}V - \frac{1}{6}\frac{4-5\delta-\alpha}{1-\delta}V)}{V} = \frac{1}{18}\frac{22 - 23\delta - \alpha}{1-\delta}$$

This is positive if $22 - 23\delta - \alpha > 0$, which is equivalent to $\alpha < 22 - 23\delta$. As $4 - 5\delta - (22 - 23\delta) = -18(1-\delta) < 0$, this is always true. ■

Proof of Corollary 1:

Recall from the proof of Proposition 3 that the consumer surplus loss is

$$-\Delta cs = -(4 - \alpha - 5\delta)\frac{5\alpha + 349\delta - 180\delta^2 - 164}{648(1-\delta)^2}.$$

With respect to α, we have

$$\frac{\partial(-\Delta cs)}{\partial\alpha} = \frac{5\alpha + 187\delta - 90\delta^2 - 92}{324(1-\delta)^2}$$

This derivative is negative if $\alpha < -\frac{187\delta - 90\delta^2 - 92}{5} = 18\delta^2 - \frac{187}{5}\delta + \frac{92}{5}$, which is implied by $\alpha < 4 - 5\delta$

because

$$\left(18\delta^2 - \frac{187}{5}\delta + \frac{92}{5}\right) - (4 - 5\delta) = \frac{18}{5}(4 - 5\delta)(1 - \delta) > 0$$

With respect to δ, we have

$$\frac{\partial(-\Delta cs)}{\partial \delta} = \frac{5\alpha^2 + \alpha(7\delta + 3) + 450\delta^3 - 1350\delta^2 + 1357\delta - 452}{324(1-\delta)^3}$$

The numerator, $5\alpha^2 + \alpha(7\delta + 3) + 450\delta^3 - 1350\delta^2 + 1357\delta - 452$, is a U-shaped parabola in α with roots

$$\begin{aligned}
\alpha^- &= -\frac{1}{10}\sqrt{9049 - 9000\delta}\,(1-\delta) - \frac{7}{10}\delta - \frac{3}{10} < 0 \\
\alpha^+ &= \frac{1}{10}\sqrt{9049 - 9000\delta}\,(1-\delta) - \frac{7}{10}\delta - \frac{3}{10}
\end{aligned}$$

The numerator is negative because as

$$\alpha^+ - (4 - 5\delta) = \frac{(1-\delta)}{10}\left(\sqrt{43^2 + 1800(4 - 5\delta)} - 43\right) > 0.$$

it is always true that $0 < \alpha < 4 - 5\delta < \alpha^+$. ■

Proof of Lemma 8:

i) A higher A increases the estimated consumer loss:

$$\frac{\partial\left(-\frac{\Delta CS}{\Theta}\right)}{\partial A} = -\frac{1}{8}(1 - s_G)\frac{20\delta + 5s_G - 21}{4\delta + 9s_G(1-\delta) - 5} > 0$$

if $20\delta + 5s_G - 21 < 0 \Leftrightarrow \delta < \frac{21-5s_G}{20}$, which is implied by $\delta < \frac{5-9s_G}{4-9s_G}$ because $\frac{5-9s_G}{4-9s_G} - \frac{21-5s_G}{20} = -\frac{1}{20}(45s_G + 16)\frac{1-s_G}{9s_G-4} < 0$.

ii) A lower s_G increases the estimated consumer loss:

$$\frac{\partial\left(-\frac{\Delta CS}{\Theta}\right)}{\partial s_G}=-\frac{1}{8}A\frac{-165\delta+50s_G-40\delta s_G+100\delta^2+45\delta s_G^2+59-45s_G^2}{\left(-4\delta-9s_G+9\delta s_G+5\right)^2}<0$$

The numerator is a U-shaped parabola in δ with roots

$$\delta^-=-\frac{1}{40}\left(-8s_G+9s_G^2-33+\sqrt{18s_G+81s_G^2+145}\,(1-s_G)\right)$$
$$\delta^+=-\frac{1}{40}\left(-8s_G+9s_G^2-33-\sqrt{18s_G+81s_G^2+145}\,(1-s_G)\right)$$

As the maximum admissible value for δ is lower than δ^-, the numerator is always positive:

$$\frac{5-9s_G}{4-9s_G}-\left(-\frac{1}{40}\left(-8s_G+9s_G^2-33+\sqrt{18s_G+81s_G^2+145}\,(1-s_G)\right)\right)$$
$$=-\frac{1}{40}(1-s_G)\frac{81s_G^2+68-27s_G-\sqrt{(81s_G^2+68-27s_G)^2-144\,(45s_G+16)}}{9s_G-4}<0$$

iii) A higher δ increases the estimated consumer loss.

$$\frac{\partial\left(-\frac{\Delta CS}{\Theta}\right)}{\partial\delta}=\frac{A\,(45s_G+16)\,(1-s_G)^2}{8\left(-4\delta-9s_G+9\delta s_G+5\right)^2}>0.$$

Appendix B

In this appendix we consider the possibility, as in Etro and Caffarra (2017, page 306), that in the absence of tying, firm G "is committed to giving the GP/GPS suite" (software X) "for free, and this is used by all OEMs since it is superior to any alternative: from this Google gains α from in-app revenues."

If this is the case, users select OEM 1 if

$$R + R_x + \delta'\Theta - P_1 > R + R_x + \delta'\Theta - P_2 \Leftrightarrow P_1 < P_2$$

As all equipment includes software X and equally valued versions of software Y, users select the one with the lowest price. The profit of the two OEM's if OEM 1 has accepted b_i and OEM 2 has accepted b_j is

$$\frac{\Pi_1}{V} = (p_1 - c - b_i)D_1$$
$$\frac{\Pi_2}{V} = (p_2 - c - b_j)D_2$$

Price competition will lead prices p_1 and p_2 to be slightly below the marginal cost of the firm that accepted the highest bid, with the OEM that accepted the lowest bid selling to all users. Both software developers will compete for the lowest bid and, in equilibrium, $b_G = b_E = -\beta$.

So, in equilibrium, $p_1 = p_2 = c - \beta$ and

$$\pi_1 = 0,\ \pi_2 = 0,\ \pi_E = 0,\ \pi_G = \alpha$$

Consumer surplus is

$$CS = (R - P_i)\,\Theta + \int_0^V \left(R_x + \delta'\Theta\right) \frac{\Theta}{V} dR_x$$
$$\frac{CS}{V\Theta} = (r - p_i) + \int_0^V \left(\frac{R_x}{V} + \delta\right) \frac{1}{V} dR_x$$
$$cs = r - c + \beta + \frac{1}{2} + \delta$$

In this scenario, the change in consumer surplus due to tying is

$$r-c+\beta+\frac{360\delta^3+8\delta^2(9\alpha-124)-8\delta(19\alpha-112)+\alpha^2+82\alpha-263}{162(1-\delta)^2}-\left(r-c+\beta+\frac{1}{2}+\delta\right)$$

The change in consumer surplus is lower in this case (i.e. the consumer loss is larger) if and only if

$$\frac{\alpha^2+\alpha\left(72\delta^2-152\delta+82\right)+198\delta^3-749\delta^2+896\delta-344}{162(1-\delta)^2}<(4-\alpha-5\delta)\frac{5\alpha+349\delta-180\delta^2-164}{648(1-\delta)^2}\Leftrightarrow$$
$$\frac{\alpha^2+\alpha\left(12\delta^2-26\delta+16\right)-12\delta^3-59\delta^2+152\delta-80}{72(\delta-1)^2}<0$$

The numerator is a U shaped parabola with roots

$$\alpha^-=13\delta-6\delta^2-8-6(1-\delta)\sqrt{4-2\delta+\delta^2}<0$$
$$\alpha^+=13\delta-6\delta^2-8+6(1-\delta)\sqrt{4-2\delta+\delta^2}$$

with $\alpha^+>4-5\delta$ because

$$13\delta-6\delta^2-8+6(1-\delta)\sqrt{4-2\delta+\delta^2}-(4-5\delta)=6(1-\delta)\left(\sqrt{(2-\delta)^2+2\delta}-(2-\delta)\right)>0$$

Thus, for all $0<\alpha<4-5\delta<\alpha^+$ the numerator is negative.

Tying is profitable in this case if and only if

$$\frac{1}{27}\frac{(5+\alpha-4\delta)^2}{1-\delta}>\alpha\Leftrightarrow\frac{\alpha^2+\alpha(19\delta-17)+16\delta^2-40\delta+25}{27(1-\delta)}>0$$

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